NEGATIVE INTEREST RATES – POSSIBLE CALIBRATION SOLUTIONS

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AGENDA

- **Interest rate risk**
  - **Current practice**
  - **What is the problem? – negative interest rates**
  - **Possible remediation – “Threshold” model**
- Interest-rate volatility risk
  - Current practice
  - What is the problem? – undefined Black volatilities
  - Possible remediation – Normal volatilities
- Questions

INTEREST RATE RISK - CURRENT PRACTICE

- In most cases, interest rate risk is usually modelled by using **principal components** of yield curve movement (spot or forward rates). *WLOG we discuss spot rates in the remainder if the presentation.*
- In particular, in the environment where spot rates are assumed positive, a model based on logarithmic changes in spot rates is often used.
- When using a **logarithmic model** of interest rate movements, historical interest-rate movements between times *t*-1 and *t* (for spot-rate maturity *n*) are expressed as follows:

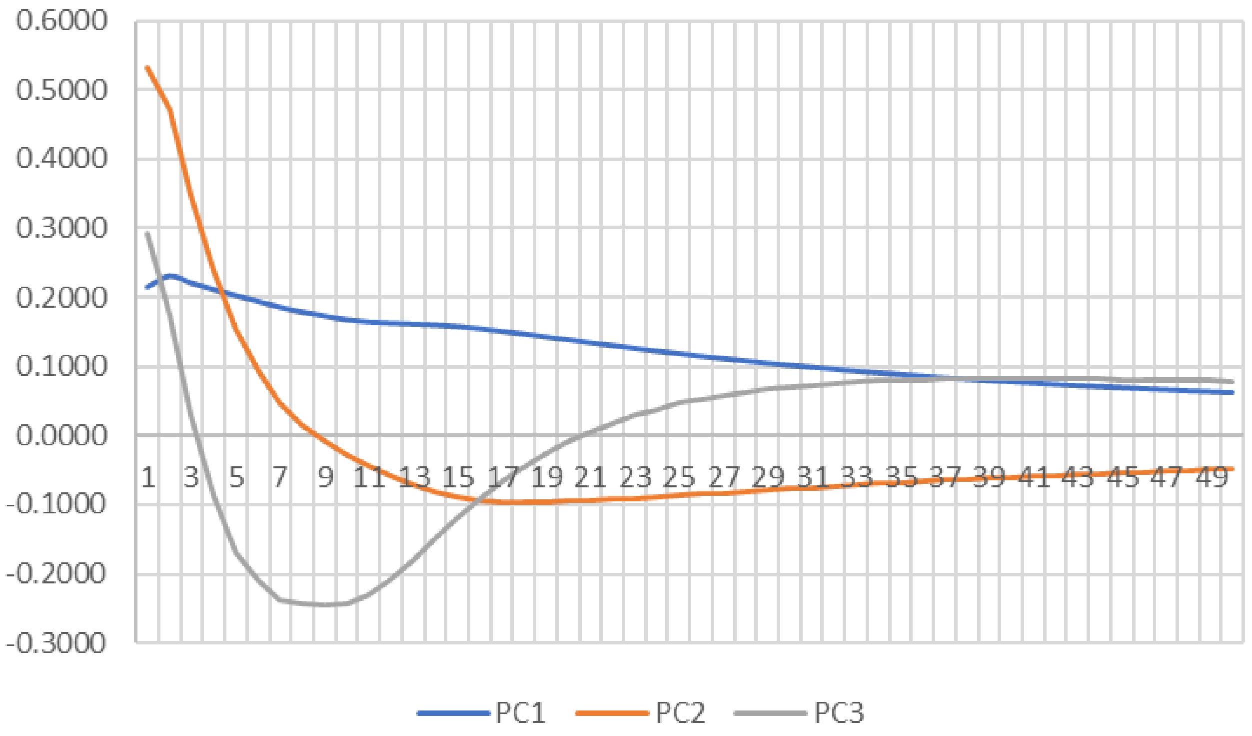
$$\varepsilon(t,n) = ln\left(\frac{R(t,n)}{R(t-1,n)}\right)$$

where $R(t,n)$ is the spot rate at time *t* for maturity *n* years.

- This setup is consistent with logarithmic (Black) formula for the pricing bond options. It also used to be seen as having a **desirable** feature of keeping interest rates positive!!!
- In some applications, a regime-based model is adopted switching between **additive** (for **high-rate** environment) and logarithmic (for **low-rate** environment).

INTEREST RATE RISK - CURRENT PRACTICE

- Principal-components analysis is then performed on historical values of $\varepsilon(t, n)$ in order to reduce the dimensionality of the analysis down to 3 or 4 risk factors.
- First three principal components typically have the following shapes (PC1 = "level", PC2 = "slope" and PC3 = "twist") reflective the eigenvectors of the covariance (or correlation) matrix of $\varepsilon(t, n)$:



INTEREST RATE RISK - CURRENT PRACTICE

- In the multi-scenario simulation (such as used in Solvency II internal model), the simulated **scenario shocks** to the initial spot-rate curve are then modelled using the following formula (or similar):

$$S(n,i) = S(n,0)e^{EV_1(n)PC_1(i)+EV_2(n)PC_2(i)+EV_3(n)PC_3(i)}$$

where

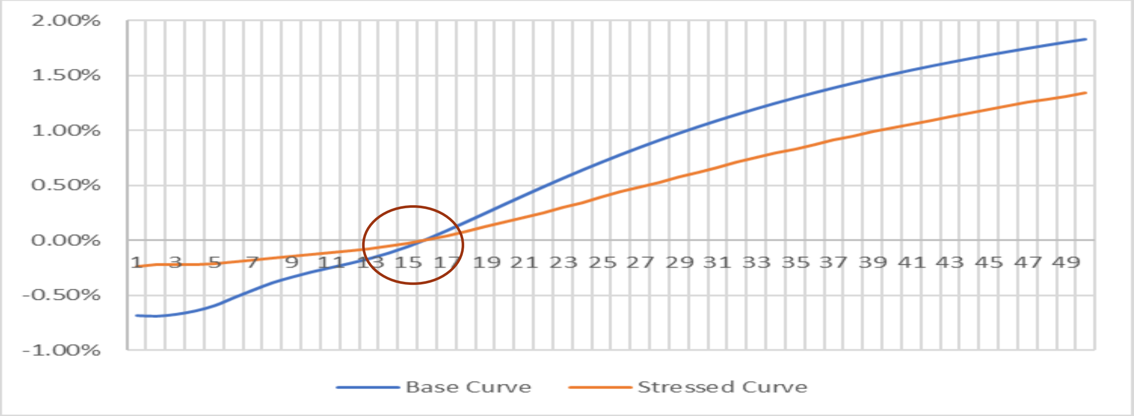
- $S(n,i)$ is the shocked spot rate for maturity n in scenario i;
- $S(n,0)$ is the base-case spot rate for maturity n;
- $EV_j(n)$ is the element of j-th eigenvector for maturity n;
- $PC_j(i)$ is the realisation for j-th PC risk factor in scenario i;

- PC risk factor are simulated in accordance with the **calibrated distributions**.
- Shocked spot rate curves are then used as inputs into the calculation of asset and liability stresses which, in turn, give the impact of interest-rate risk on the NAV.

INTEREST RATE RISK – WHAT IS THE PROBLEM?

- **Firstly,** the presence of **spot rates switching signs** makes the calculation of historical logarithmic interest-rate movements impossible:
  - $\varepsilon(t,n) = ln\left(\frac{R(t,n)}{R(t-1,n)}\right)$ is **undefined** when $R(t,n)$ and $R(t-1,n)$ are of a different sign
  - Historical observations containing sign-switch for any maturity must therefore be discarded from PCA and from the fitting of PC risk factor distributions

- **Secondly,** even if the undesirable observations can be discarded, the resulting PC1, PC2 and PC3 shocks have shapes that no longer have intuitive economic interpretation.
  - Recall the formula for the shocked spot rate: $S(n,i) = S(n,0)e^{EV_1(n)PC_1(i)+EV_2(n)PC_2(i)+EV_3(n)PC_3(i)}$
  - This formula assumes preservation of the sign of spot rate
  - This results in, for example, the "level" PC1 shock having the following shape (resembling PC2 "slope" shock):



- **Thirdly,** as can be seen from the graph above, the spot rates that are **at (or close to) zero** remain almost **unchanged** in any shock scenario (also as a consequence of the shocked curve formula)

INTEREST RATE RISK – WHAT CAN WE DO?

- The task at hand is two-fold:
  - Re-define the formula $\varepsilon(t,n) = ln\left(\frac{R(t,n)}{R(t-1,n)}\right)$ so as to incorporate the possibility of **sign change**…
  - … at the same time preserving **control** over the extent of **negative the spot rates** can be
- The latter objective ensures
  - Avoidance of economically unfeasible scenarios
  - Importantly avoids the possibility of overly strenuous scenarios for entities strongly exposed to downward interest-rate shock.
- Two most common options are:
  - **Additive** model: $\varepsilon(t,n) = R(t,n) - R(t-1,n)$
  - **Threshold-based** logarithmic model: $\varepsilon(t,n) = ln\left(\frac{R(t,n) - \alpha}{R(t-1,n) - \alpha}\right)$, where $\alpha<0$ is the "interest-rate threshold", i.e. the level below which interest rates cannot go in ANY scenario
- The Additive model allows **no control over the extent of downward shocks** (unless an explicit floor is imposed)
- **Threshold-based** model is therefore usually preferred

INTEREST RATE RISK – WHAT CAN WE DO?

- Under the threshold-based model, the formula for simulated shocked spot-rate curve changes as follows:

$$(S(n,i)-\alpha)=(S(n,0)-\alpha)e^{EV_1(n)PC_1(i)+EV_2(n)PC_2(i)+EV_3(n)PC_3(i)}$$

where

- $S(n,i)$ is the shocked spot rate for maturity *n* in scenario *i*;
- $S(n,0)$ is the base-case spot rate for maturity *n*;
- $EV_j(n)$ is the element of *j*-th eigenvector for maturity *n*;
- $PC_j(i)$ is the realisation for *j*-th PC risk factor in scenario *i*;
- α is the interest-rate threshold

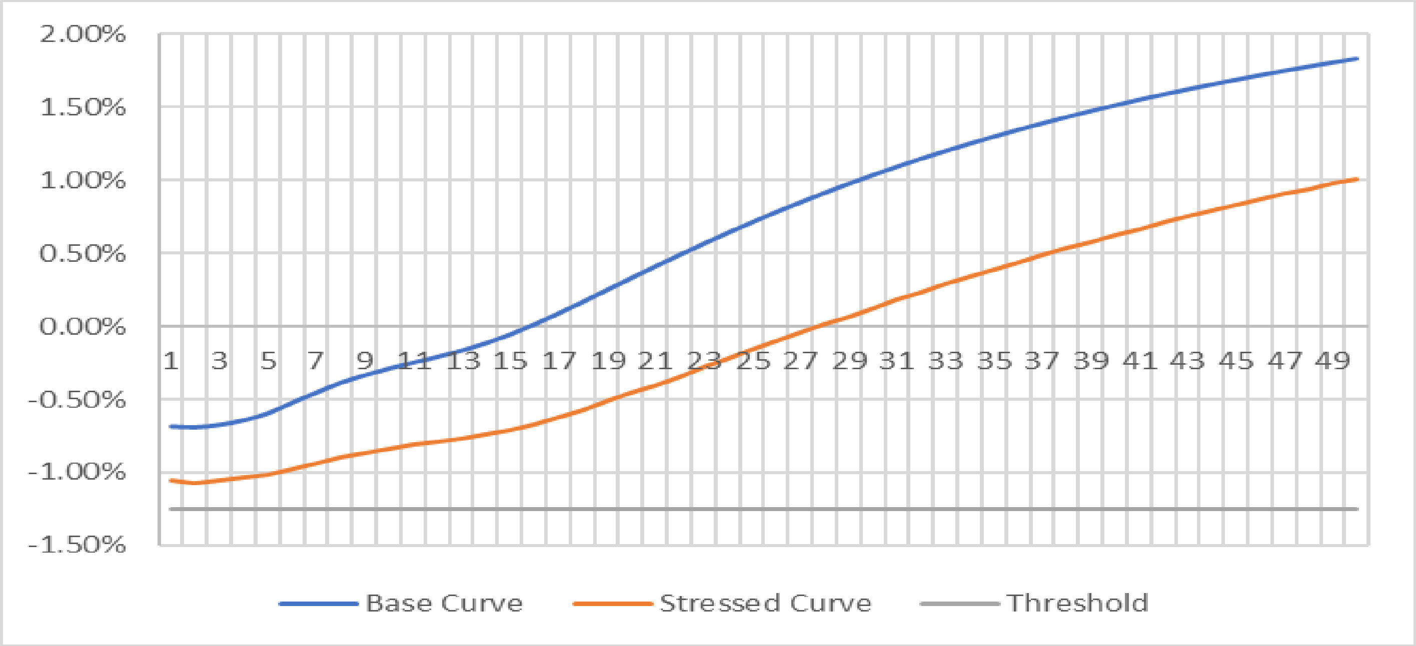
NB!!!

In this case, the PCA is performed in threshold adjusted logarithmic interest-rate movements:

$$\varepsilon(t,n)=ln\left(\frac{R(t,n)-\alpha}{R(t-1,n)-\alpha}\right)$$

INTEREST RATE RISK – HOW DOES THIS HELP?

- **Firstly,** the issue of **spot rates switching signs** no longer exists:
  - $\varepsilon(t,n) = ln\left(\frac{R(t,n) - \alpha}{R(t-1,n) - \alpha}\right)$ is **always defined** if the threshold is "sufficiently low" (more on this below)
- **Secondly,** provided the threshold is set "sufficiently low", the resulting PC shocks preserve the intuitive economic interpretation (level, slope and twist):
  - For example, the "level" PC1 shock appears as follows:



- **Thirdly,** as can be seen from the graph above, the spot rates that are **at (or close to) zero** are still subject to shocks

INTEREST RATE RISK – THRESHOLD

- The value of threshold is not an objective number and is a matter of economic and actuarial judgment. The following considerations will influence the choice of threshold:
  1. The value has to be "sufficiently low" to remove the negative argument in the logarithmic function, i.e. **lower than any historically observed spot rate for ANY maturity**. This is also intuitively obvious (if a spot rate of a certain negative values has occurred in the past, it is hard to argue that it will not re-occur).
  2. A threshold **just below historical minimum observed** spot rate (e.g. 1bp or 5bps below) will lead to volatile values of $\varepsilon(t,n)$ and **instability of PCA** which results in problems for distribution fitting.
  3. It is generally desirable to ensure that the threshold is changed very infrequently (preferably never) as the underlying judgment is that **"spot rates will never cross the threshold".** Furthermore, depending on the internal model setup, changing the level of threshold may complicate the analysis-of-change process and may require **explicit regulatory approval**.
  4. Very **little historical data** of deep negative rates is available in most currencies, with the exception of Swiss franc, where spot rates of almost -1% have recently been observed for short maturities (<5 YRS).
  5. Sensitivity analysis of SCR to the choice of threshold is vital.
     1. Remember that threshold is the lowest possible value.
     2. 50bps change in threshold does not mean 50bps change in a 1:200 shock (usually much less). The consequential effect on asset values may therefore not be very dramatic.
     3. After the effect of **LAC and diversification**, the effect is likely to be even more **diluted**.

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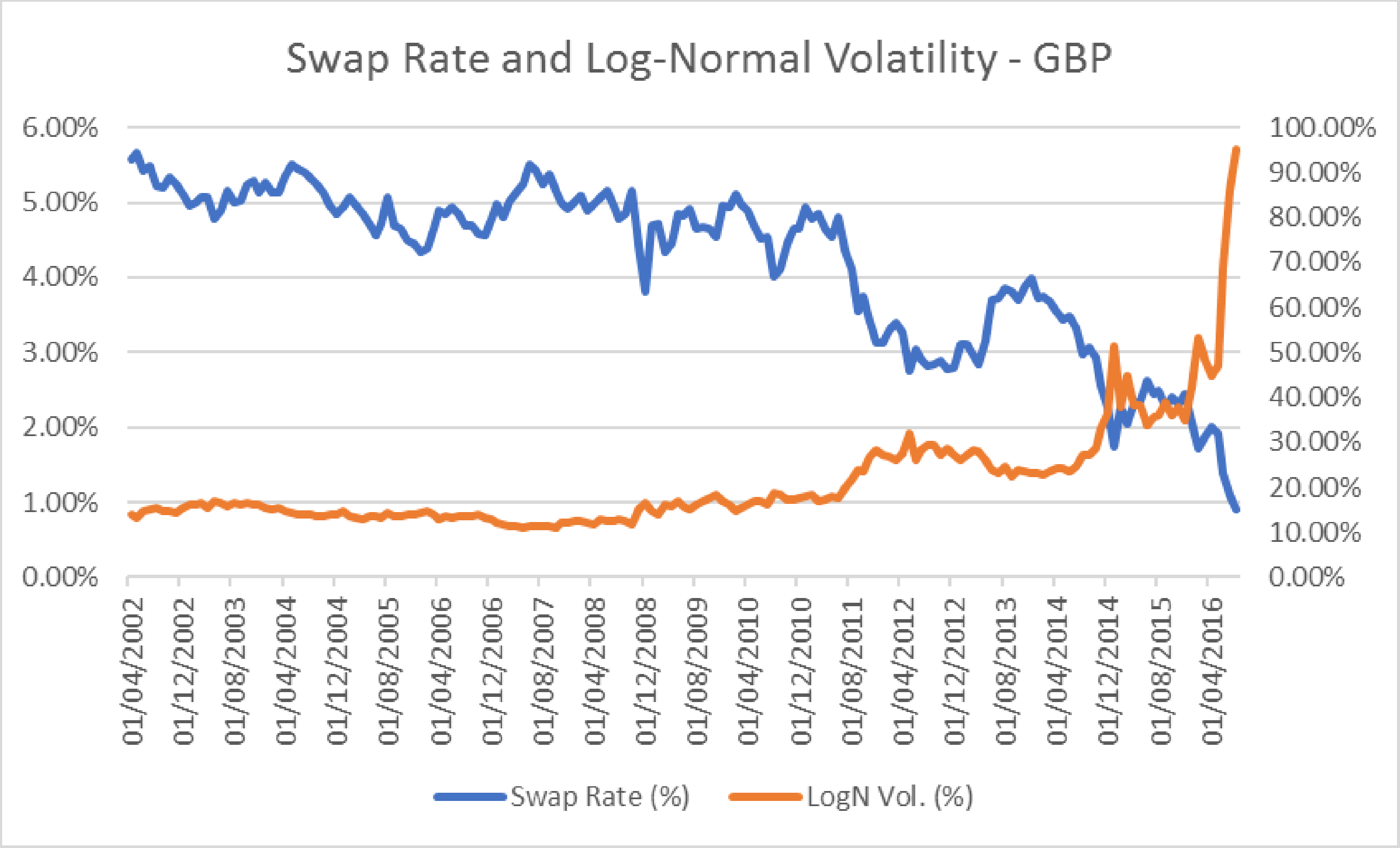
IR VOLATILITY RISK- CURRENT PRACTICE

- The most common measure of interest rate volatility used recently in internal-model calibrations is the **Black swaption-implied volatility**.
- This represents the implied volatility obtained by inverting the Black (1976) swaption-pricing formula, which is based on an assumption of **swap rates** being **log-normally distributed**
- Importantly, Black volatility is:
  - **Proportional** to the value of swap rate, and
  - Based on the assumption of lognormal distribution of swap rates, i.e. assuming that **swap rates always remain positive**
- When swap rates are **negative** (and, in fact, for some positive values of swap rates – *see below*), the Black swaption-implied volatility is **not defined**

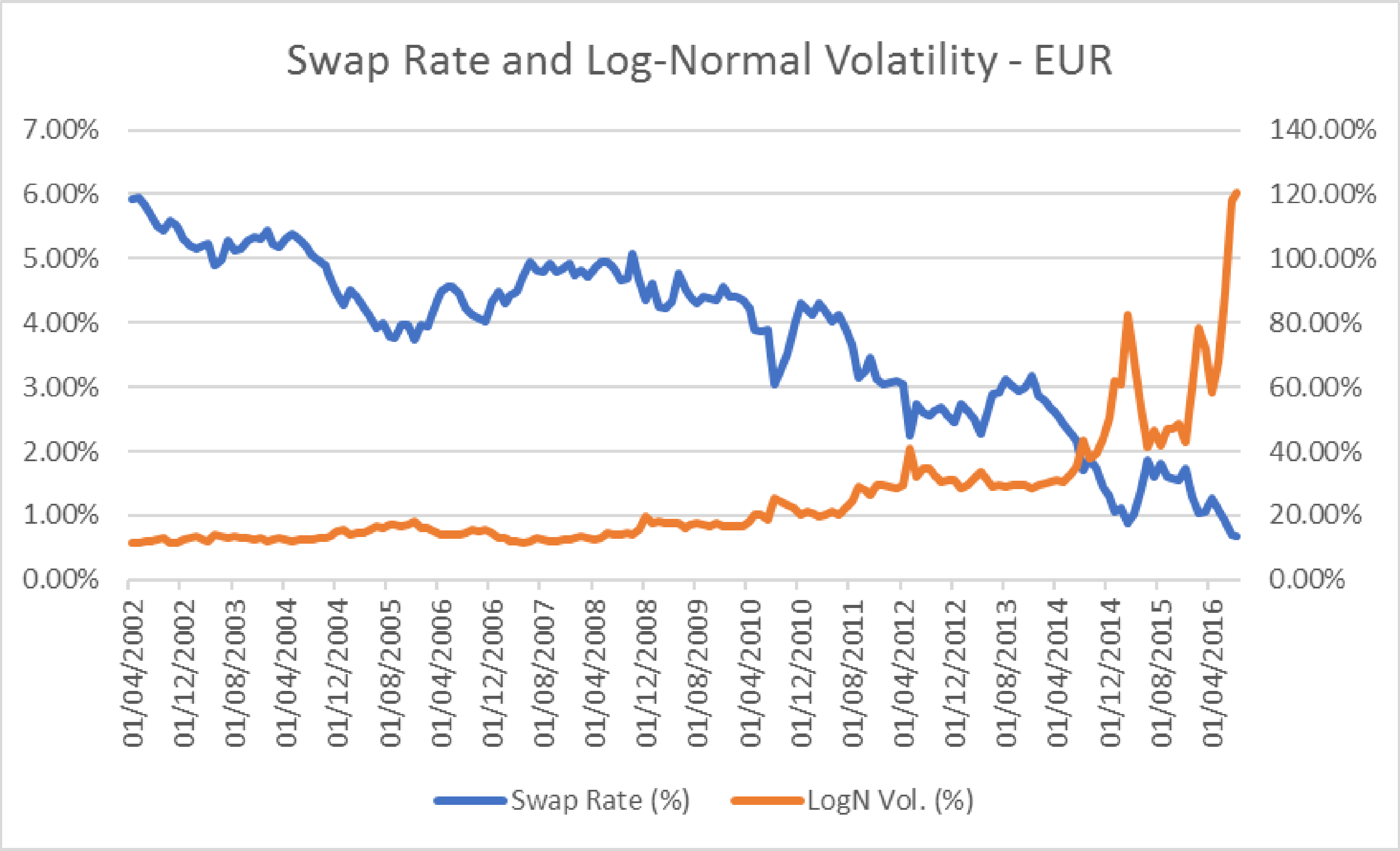
IR VOLATILITY RISK – WHAT IS THE PROBLEM?

- GBP swap rates and **Log-Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



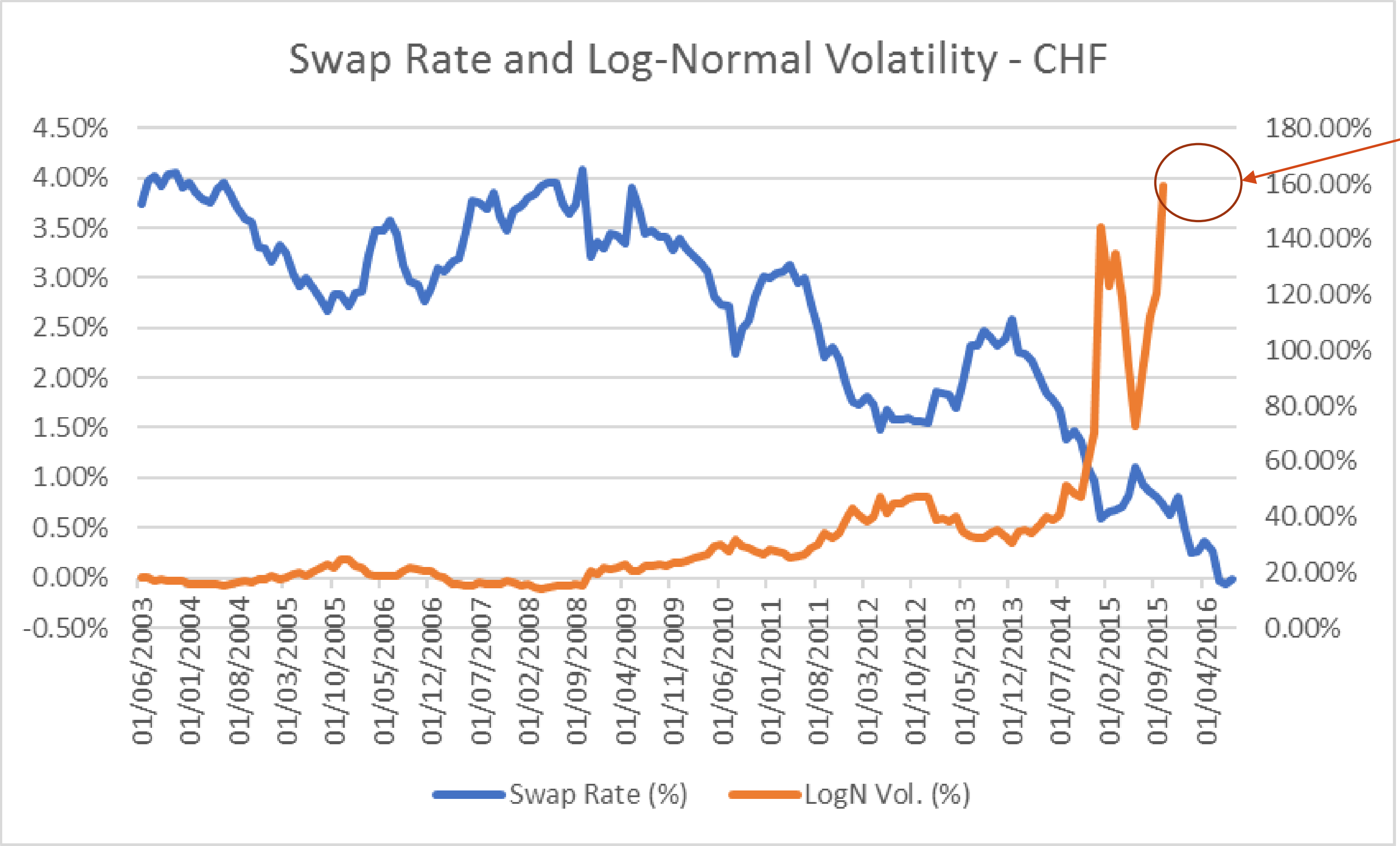
IR VOLATILITY RISK – WHAT IS THE PROBLEM?

- EUR swap rates and **Log-Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



IR VOLATILITY RISK – WHAT IS THE PROBLEM?

- CHF swap rates and **Log-Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



From here onwards, the LogN implied volatilities either explode into 1000's of % or are undefined.

IR VOLATILITY RISK – WHAT IS THE PROBLEM?

- Exploding values of Black implied volatilities complicate the calibration of volatility risk factors:

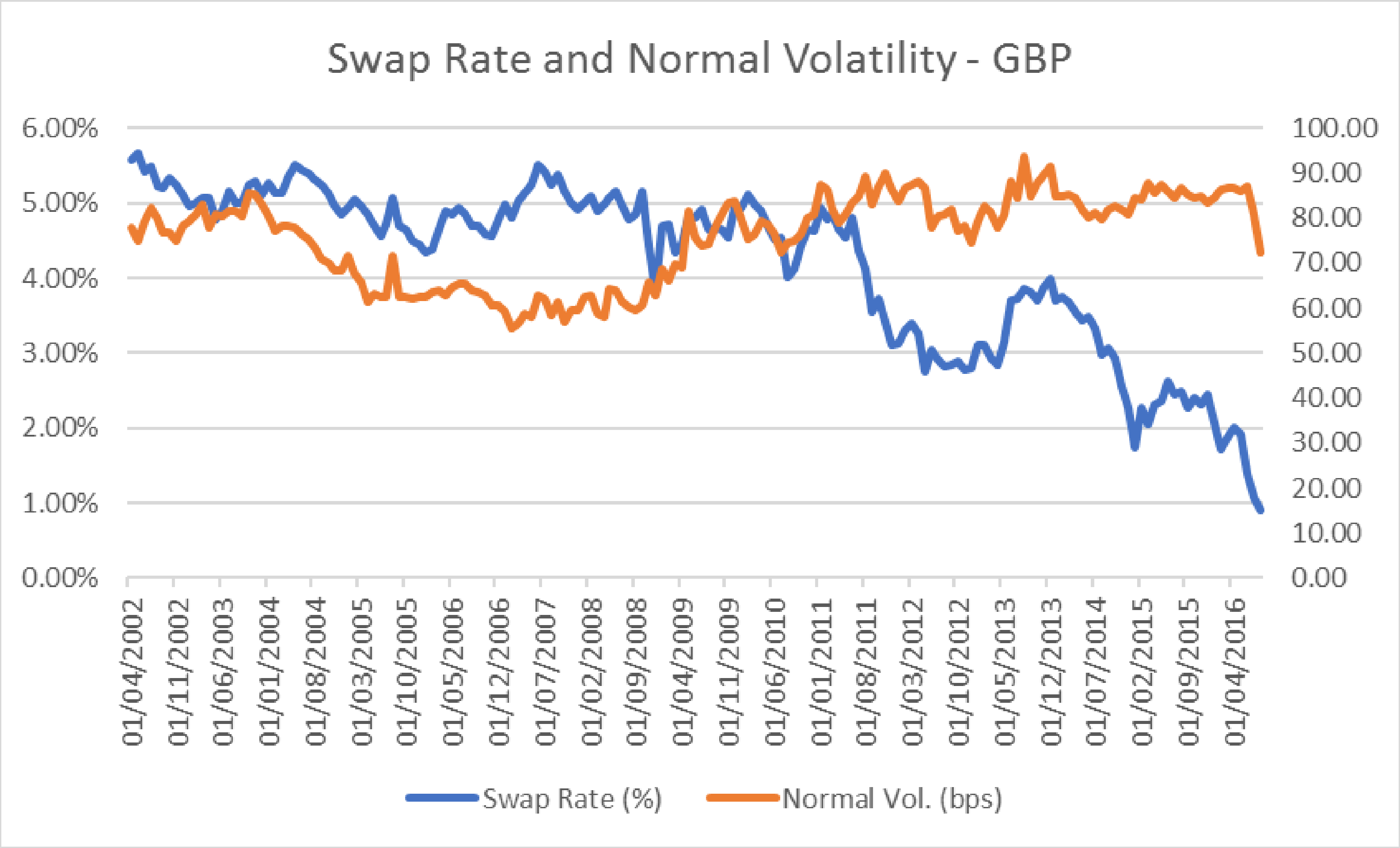
1. Implied volatility time series may contain **undefined (#N/A) values** due to the presence of negative swap rates (already seen for EUR and CHF)
2. Using an additive model of implied-volatility shock may lead to a high probability of negative implied volatilities (due to a large spread of potential values)
3. Using a multiplicative (or logarithmic) of implied-volatility shocks creates a procyclical effect – high starting values of implied volatility lead to **extremely high upward shocks**.
4. This has a potential to produce unrealistic shocked values that imply unrealistically onerous shocks on **volatility-sensitive liabilities** (e.g. GAOs).
5. At very low levels of interest rate, the volatility shock is very highly negatively correlated with interest-rate shock – this may create:
   1. **PSD problems** for the correlation matrix (especially if its dimensionality is high), and
   2. Large SCR contribution from **cross-product** terms

IR VOLATILITY RISK – WHAT CAN WE DO?

- A different measure of interest-rate volatility is needed which has the following properties:
  - **Looser dependency** on the value of interest rates
  - More **stable values over time** (not exploding or collapsing by an order of magnitude)
  - **Always defined**, irrespective of positive or negative interest rates
  - Widely quoted by respectable data providers
- The most obvious candidate is **Normal implied volatility** which is solved from the Bachelier (1900) option pricing formula - a predecessor to Black formula which assumed a Normal underlying distribution (s opposed to Log-Normal).
- All of the above properties are satisfied by Normal implied volatility. Normal volatility surface is currently quoted by both Bloomberg and SuperDerivatives.

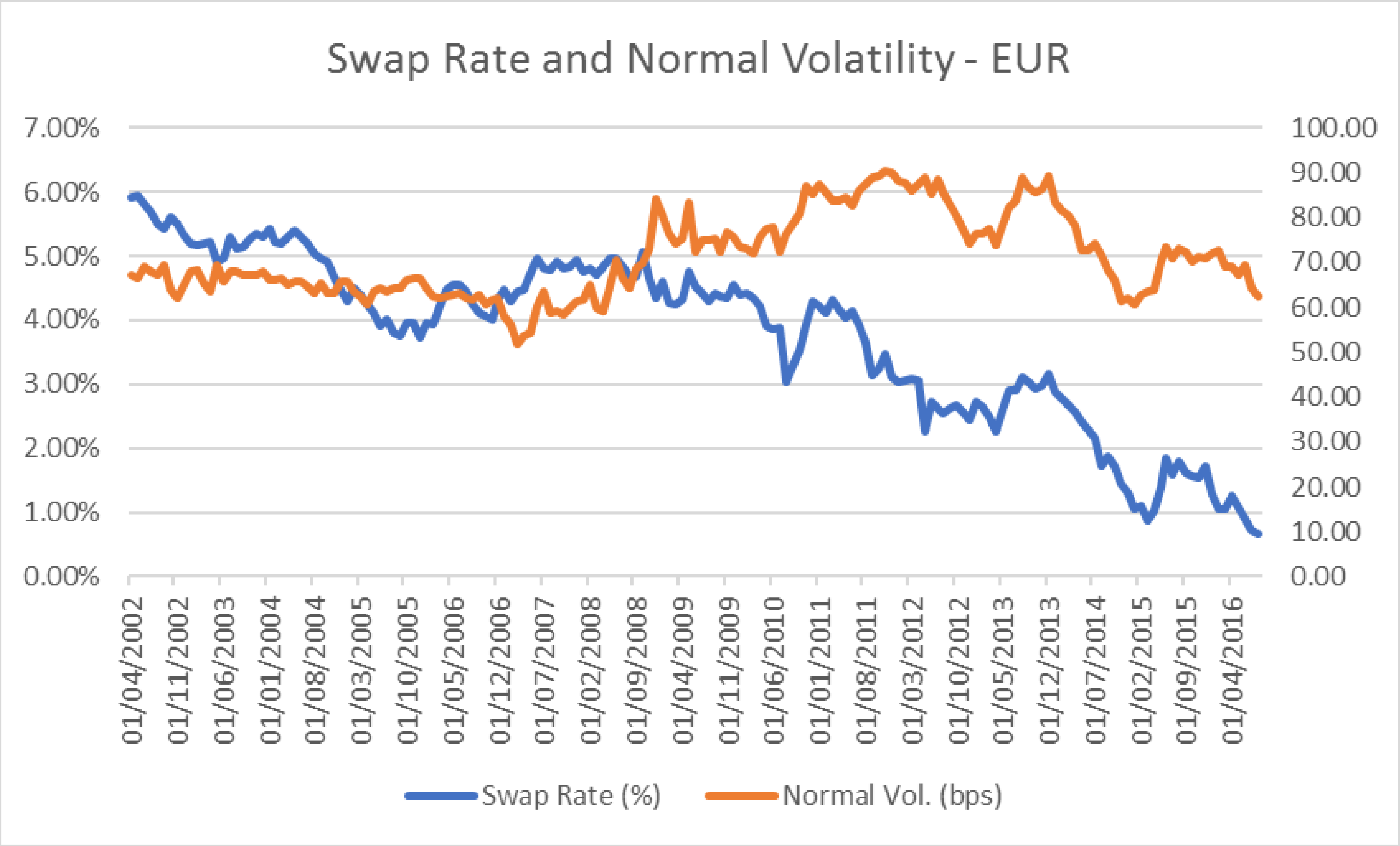
IR VOLATILITY RISK – HOW DOES THIS HELP?

- GBP swap rates and **Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



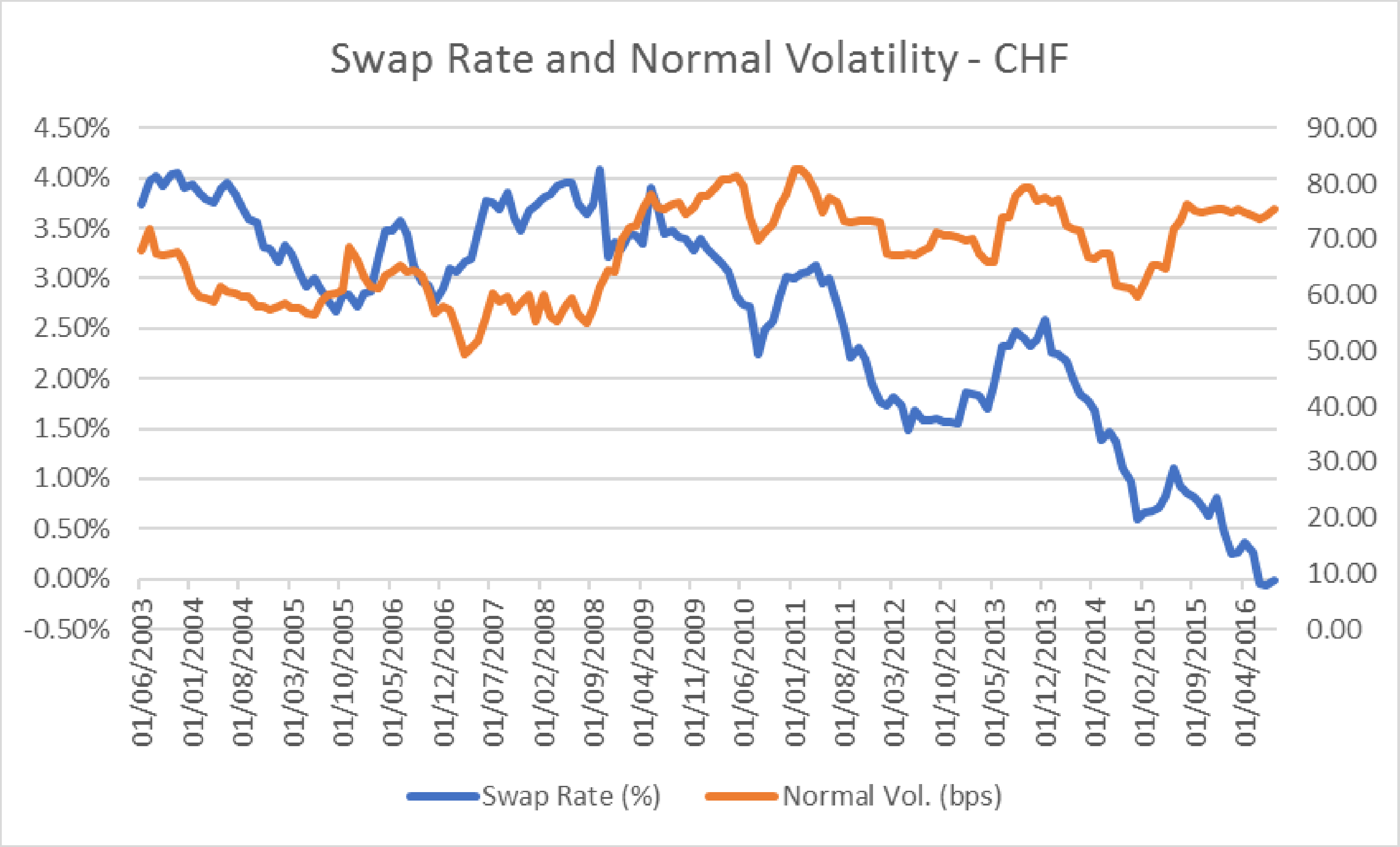
IR VOLATILITY RISK – HOW DOES THIS HELP?

- EUR swap rates and **Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



IR VOLATILITY RISK – HOW DOES THIS HELP?

- CHF swap rates and **Normal** implied volatilities for swap rates and swaption implied volatilities (option maturity = 5 years, swap rate = 5 years):



IR VOLATILITY RISK – HOW DOES THIS HELP?

	Log-Normal (Black) Vol.	Normal Vol.
Correlation with IR level	Between -60% and -90%	Between -25% and +25%
Historical range	Between 10% and >100% (order of magnitude)	Between 50bps and 100bps
Definition	Logarithmic - undefined for very low and negative swap rates	Absolute - always defined

IR VOLATILITY RISK – WHEN IS LOG-NORMAL UNDEFINED?

- Normal volatility **always exists** as it is the implied volatility of **absolute movement** of swap rate (not a logarithmic volatility)

- An approximate (almost exact) relationship between Normal implied volatility $V_N(m,k)$ and Log-Normal (Black) implied volatility $V_{LN}(m,k)$ is as follows:

$$\boldsymbol{V_N(m,k) \approx \frac{V_{LN}(m,k)s(m,k)}{1+\gamma m {V_{LN}}^2(m,k)}}$$

where

  - m is the maturity of the option
  - k is the tenor of the swap
  - $s(m,k)$ is the ATM forward swap rate
  - $\gamma = 1/24$

- Conversely, the above formula can be solved for $V_{LN}(m,k)$:

$$\boldsymbol{V_{LN}(m,k) \approx \frac{s - \sqrt{s^2(m,k) - 4\gamma m {V_N}^2(m,k)}}{2\gamma m V_N(m,k)} = \frac{s - \sqrt{s^2(m,k) - m{V_N}^2(m,k)/6}}{mV_N(m,k)/12}}$$

- Note that it is the **subtractive** (and not additive) root of the quadratic equation that gives the correct Log-Normal volatility.

- Note also that **Log-Normal volatility is only defined if the discriminant is not negative**, i.e.

$$s^2(m,k) \geq 4\gamma m {V_N}^2(m,k) = m{V_N}^2(m,k)/6$$

THANK YOU!!!